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TOWARD A MORE UNIFIED THEORY OF MONOPULSE

RHODE'S THEORY OF MONOPULSE - REVISITED

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A modification to Rhodes' Theory of Monopulse is proposed, and then used to analyze the theory behind an "amplitude phase interferometer" (p. 98, Microwave Journal, Sep 1983).

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PREFACE

The author's background is neither in sensors nor electrical engineering but in simulation and physics. So in effect this report is a view of monopulse from an outsider looking in. It is hoped it may be of help to other outsiders, and may be of interest to monopulse insiders.

The author wishes to thank insiders, Larry McWhorter of Computer Sciences Corporation for his comments at an early stage, and Dwight McPherson of Simulation Technology for his at a later stage, in the development of this outsider's view.

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I. INTRODUCTION

Pournelles' Law: "Iron is expensive, but silicon is cheap."

In many analysis of monopulse radar sensors small angles are implicitly assumed. The reason for this is that the sensor information is intended as an error signal to drive the antenna gimbles to point the antenna at the target. The gimble rates may in turn be used to command a missile to a collision course with the target via proportional navigation.

Proportional navigation intercept guidance laws may be derived from optimal intercept guidance laws by assuming constant speed and small angles.

If body fixed monopulse sensor signals are to be used in a bearing only tracking filter to estimate the state, and that state then used in a optimal intercept guidance law, small angle assumptions are no longer warranted.

With our rapidly changing technology, assumptions made some decades ago may no longer be necessary and could result in our not taking full advantage of present technology.

II. TOWARD A MORE UNIFIED THEORY OF MONOPULSE

Monopulse is one form of radio direction finding. The goal of monopulse or simultaneous lobe comparison is to reduce or remove the effects of the variation of the received signal with time in determining the direction of arrival of the received signal.

Assume the receiving antenna's lobe pattern is a complex exponential function of the direction of arrival,

$$A = G\, e^{\psi(u) + i\phi(u)} \quad , \tag{1}$$

where G and $\psi(u)$ determine the amplitude and $\phi(u)$ determines the phase, and u is the direction of arrival. Recall that a function is composed of even and odd components, that is,

$$\psi(u) = \psi_e(u) + \psi_o(u) \quad , \tag{2}$$

and

$$\phi(u) = \phi_e(u) + \phi_o(u) \quad . \tag{3}$$

The other lobe pattern is

$$B = G\, e^{\psi(-u) + i\phi(-u)} \quad , \tag{4}$$

where

$$\psi(-u) = \psi_e(u) - \psi_o(u) \quad , \tag{5}$$

and

$$\phi(-u) = \phi_e(u) - \phi_o(u) \quad . \tag{6}$$

Rhodes assumed

$$A = P(u)\ e^{i\phi(u)} \quad , \tag{7}$$

and

$$B = P(-u)\ e^{i\phi(-u)} \quad , \tag{8}$$

and it follows that

$$P(u) = G\ e^{\psi(u)} \quad , \tag{9}$$

and

$$P(-u) = G\ e^{\psi(-u)} \quad . \tag{10}$$

The ratio is

$$\frac{A}{B} = \frac{G\ e^{\psi_e(u) + i\phi_e(u)}\ e^{\psi_o(u) + i\phi_o(u)}}{G\ e^{\psi_e(u) + i\phi_e(u)}\ e^{-\psi_o(u) - i\phi_o(u)}} \quad , \tag{11}$$

$$\frac{A}{B} = e^{2(\psi_o(u) + i\phi_o(u))} \quad . \tag{12}$$

Similarly

$$\frac{A - B}{A + B} = \frac{G\ e^{\psi_e(u) + i\phi_e(u)}\left(e^{\psi_o(u) + i\psi_o(u)} - e^{-\psi_o(u) - i\phi_o(u)}\right)}{G\ e^{\psi_e(u) + i\phi_e(u)}\left(e^{\psi_o(u) + i\psi_o(u)} + e^{-\psi_o(u) - i\phi_o(u)}\right)} \quad , \tag{13}$$

$$\frac{A - B}{A + B} = \tanh(\psi_o(u) + i\phi_o(u)) \quad , \tag{14}$$

$$\frac{A - B}{A + B} = \frac{\sinh(2\psi_o(u)) + i\ \sin(2\phi_o(u))}{\cosh(2\psi_o(u)) + \quad \cos(2\phi_o(u))} \quad . \tag{15}$$

Note that, equations (12, 14),

$$1/2\ \mathrm{Ln}\left(\frac{A}{B}\right) = \mathrm{arctanh}\left(\frac{A - B}{A + B}\right) \quad , \tag{16}$$

a well known identity. Another point to note is that in equations (11, 13) the even components factor out and cancel out, and the monopulse information is contained in the odd components.

III. A B AMPLITUDE MONOPULSE

Representing A and B as two dimensional vectors instead of complex variables, one has

$$A = G\, e^{\psi(u)} \begin{pmatrix} \cos\phi(u) \\ \sin\phi(u) \end{pmatrix} \quad , \tag{17}$$

and

$$B = G\, e^{\psi(-u)} \begin{pmatrix} \cos\phi(-u) \\ \sin\phi(-u) \end{pmatrix} \quad . \tag{18}$$

Taking the dot products,

$$A \cdot A = G^2\, e^{2\psi(u)} \left[\cos^2\phi(u) + \sin^2\phi(u)\right] \quad , \tag{19}$$

$$A \cdot A = G^2\, e^{2\psi(u)} \quad , \tag{20}$$

and

$$B \cdot B = G^2\, e^{2\psi(-u)} \left[\cos^2\phi(-u) + \sin^2\phi(-u)\right] \quad , \tag{21}$$

$$B \cdot B = G^2\, e^{2\psi(-u)} \quad . \tag{22}$$

The ratio of the dot products is

$$\frac{A \cdot A}{B \cdot B} = \frac{G^2\, e^{2\psi(u)}}{G^2\, e^{2\psi(-u)}} \tag{23}$$

$$\frac{A \cdot A}{B \cdot B} = e^{2(\psi(u) - \psi(-u))} \tag{24}$$

$$\frac{A \cdot A}{B \cdot B} = e^{4\psi_o(u)} \tag{25}$$

and solving

$$\psi_o(u) = 1/4 \,\mathrm{Ln} \left(\frac{A \cdot A}{B \cdot B}\right) \tag{26}$$

This report discusses what to implement and not how to implement. Envelope detectors are implicit in most of the material presented. Equation (26) implies square law detectors, or linear detectors whose outputs are squared.

Of course the magnitudes are the square root of the dot products, and taking the ratio of the magnitudes,

$$\frac{(\mathbf{A}\cdot\mathbf{A})^{1/2}}{(\mathbf{B}\cdot\mathbf{B})^{1/2}} = \frac{G\, e^{\psi(u)}}{G\, e^{\psi(-u)}} \quad , \tag{27}$$

$$\frac{(\mathbf{A}\cdot\mathbf{A})^{1/2}}{(\mathbf{B}\cdot\mathbf{B})^{1/2}} = e^{\psi(u) - \psi(-u)} \quad , \tag{28}$$

$$\frac{(\mathbf{A}\cdot\mathbf{A})^{1/2}}{(\mathbf{B}\cdot\mathbf{B})^{1/2}} = e^{2\psi_o(u)} \quad . \tag{29}$$

Solving

$$\psi_o(u) = 1/2 \;\mathrm{Ln}\left[\frac{(\mathbf{A}\cdot\mathbf{A})^{1/2}}{(\mathbf{B}\cdot\mathbf{B})^{1/2}}\right] \quad , \tag{30}$$

or

$$\psi_o(u) = 1/2 \;\mathrm{Ln}\left[(\mathbf{A}\cdot\mathbf{A})^{1/2}\right] - 1/2 \;\mathrm{Ln}\left[(\mathbf{B}\cdot\mathbf{B})^{1/2}\right] \quad , \tag{31}$$

the form of perhaps the first amplitude monopulse, which was invented by Sommers[1].

IV. A B PHASE MONOPULSE

The dot product of A and B is

$$\mathbf{A}\cdot\mathbf{B} = G^2\, e^{\psi(u) + \psi(-u)} \left[\cos\phi(u)\,\cos\phi(-u) + \sin\phi(u)\,\sin\phi(-u)\right] \quad , \tag{32}$$

$$\mathbf{A}\cdot\mathbf{B} = G^2\, e^{\psi(u) + \psi(-u)} \cos(\phi(u) - \phi(-u)) \quad , \tag{33}$$

$$\mathbf{A}\cdot\mathbf{B} = G^2\, e^{2\psi_e(u)} \cos(2\phi_o(u)) \quad . \tag{34}$$

Let

$$\mathbf{R} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad , \tag{35}$$

a rotation matrix for ninety degrees, which correspond to a phase shift of ninety degrees. Then,

$$\mathbf{A}\cdot\mathbf{RB} = G^2\, e^{\psi(u) + \psi(-u)} \left[\cos\phi(u)\,\sin\phi(-u) - \sin\phi(u)\,\cos\phi(-u)\right] \quad , \tag{36}$$

$$\mathbf{A}\cdot\mathbf{RB} = G^2\, e^{\psi(u) + \psi(-u)} \sin(\phi(u) - \phi(-u)) \quad , \tag{37}$$

$$\mathbf{A}\cdot\mathbf{RB} = G^2\, e^{2\,\psi_e(u)} \sin(2\phi_o(u)) \quad . \tag{38}$$

Taking the ratio, one has

$$\frac{\mathbf{A} \cdot \mathbf{RB}}{\mathbf{A} \cdot \mathbf{B}} = \tan(2\phi_o(u)) \quad , \tag{39}$$

and solving

$$\phi_o(u) = 1/2 \arctan\left[\frac{\mathbf{A} \cdot \mathbf{RB}}{\mathbf{A} \cdot \mathbf{B}}\right] \quad . \tag{40}$$

V. SUM DIFFERENCE MONOPULSE

Since

$$\mathbf{\Sigma} = \mathbf{A} + \mathbf{B} \quad , \tag{41}$$

and

$$\mathbf{\Delta} = \mathbf{A} - \mathbf{B} \quad , \tag{42}$$

it follows, since

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A} \quad , \tag{43}$$

that

$$\mathbf{\Sigma} \cdot \mathbf{\Sigma} = (\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} + \mathbf{B}) \quad , \tag{44}$$

$$\mathbf{\Sigma} \cdot \mathbf{\Sigma} = \mathbf{A} \cdot \mathbf{A} + 2\mathbf{A} \cdot \mathbf{B} + \mathbf{B} \cdot \mathbf{B} \quad , \tag{45}$$

and

$$\mathbf{\Delta} \cdot \mathbf{\Delta} = (\mathbf{A} - \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B}) \quad , \tag{46}$$

$$\mathbf{\Delta} \cdot \mathbf{\Delta} = \mathbf{A} \cdot \mathbf{A} - 2\mathbf{A} \cdot \mathbf{B} + \mathbf{B} \cdot \mathbf{B} \quad . \tag{47}$$

From equations (20, 22, 34, 45),

$$\mathbf{\Sigma} \cdot \mathbf{\Sigma} = G^2 \left[e^{2\psi(u)} + 2\, e^{2\psi_e(u)} \cos(2\phi_o(u)) + e^{2\psi(-u)}\right] \quad , \tag{48}$$

factoring out the even component,

$$\mathbf{\Sigma} \cdot \mathbf{\Sigma} = G^2\, e^{2\psi_e(u)} \left[e^{2\psi_o(u)} + 2\cos(2\phi_o(u)) + e^{-2\psi_o(u)}\right] \quad , \tag{49}$$

and from the definition of hyperbolic cosine,

$$\mathbf{\Sigma} \cdot \mathbf{\Sigma} = 2\, G^2\, e^{2\psi_e(u)} \left[\cosh(2\psi_o(u)) + \cos(2\phi_o(u))\right] \quad . \tag{50}$$

Similarly

$$\Delta \cdot \Delta = 2\ G\ e^{2\psi_e(u)} \left[\cosh(2\psi_o(u)) - \cos(2\phi_o(u))\right] \tag{51}$$

Taking the ratio of the magnitudes, one has

$$\frac{(\Delta \cdot \Delta)^{1/2}}{(\Sigma \cdot \Sigma)^{1/2}} = \left(\frac{\cosh(2\psi_o(u)) - \cos(2\phi_o(u))}{\cosh(2\psi_o(u)) + \cos(2\phi_o(u))} \right)^{1/2} \quad . \tag{52}$$

As with A B monopulse, the even components have vanished, but note that in this case both odd components remain.

For

$$\psi_o(u) = 0 \quad , \tag{53}$$

one has

$$\frac{(\Delta \cdot \Delta)^{1/2}}{(\Sigma \cdot \Sigma)^{1/2}} = \left(\frac{1 - \cos(2\phi_o(u))}{1 + \cos(2\phi_o(u))} \right)^{1/2} \quad , \tag{54}$$

which may be simplified to

$$\frac{(\Delta \cdot \Delta)^{1/2}}{(\Sigma \cdot \Sigma)^{1/2}} = \left| \tan\phi_o(u) \right| , \ \psi_o(u) = 0 \quad , \tag{55}$$

and finally

$$\left| \phi_o(u) \right| = \arctan \left[\frac{(\Delta \cdot \Delta)^{1/2}}{(\Sigma \cdot \Sigma)^{1/2}} \right] , \ \psi_o(u) = 0 \quad , \tag{56}$$

sum difference phase monopulse.

Similarly for

$$\phi_o(u) = 0 \quad , \tag{57}$$

equation (52) becomes

$$\frac{(\Delta \cdot \Delta)^{1/2}}{(\Sigma \cdot \Sigma)^{1/2}} = \left(\frac{\cosh(2\psi_o(u)) - 1}{\cosh(2\psi_o(u)) + 1} \right)^{1/2} \quad , \tag{58}$$

which may be simplified to

$$\frac{(\Delta \cdot \Delta)^{1/2}}{(\Sigma \cdot \Sigma)^{1/2}} = \left| \tanh\psi_o(u) \right| , \ \phi_o(u) = 0 \quad , \tag{59}$$

and solving

$$/\psi_o(u)/ = \text{arctanh}\left[\frac{(\Delta \cdot \Delta)^{1/2}}{(\Sigma \cdot \Sigma)^{1/2}}\right], \quad \phi_o(u) = 0 \quad , \tag{60}$$

sum difference amplitude monopulse. Actually equation (59) is the form usually implemented because of the sensitivity to noise of the inverse hyperbolic tangent to quantities near one.

VI. AN "AMPLITUDE PHASE INTERFEROMETER"[2]

What if neither $\psi_o(u)$ nor $\phi_o(u)$ are zero in equation (52)? What was proposed[2] was

$$\text{Ln}\left[((A - B) \cdot (A - B))^{1/2}\right] - \text{Ln}\left[((A + B) \cdot (A + B))^{1/2}\right] \quad , \tag{61}$$

$$\text{Ln}\left[((A + RB) \cdot (A + RB))^{1/2}\right] - \text{Ln}\left[((A - RB) \cdot (A - RB))^{1/2}\right] \quad , \tag{62}$$

and

$$\text{Ln}\left[(A \cdot A)^{1/2}\right] - \text{Ln}\left[(B \cdot B)^{1/2}\right] \tag{63}$$

to remove cycle ambiguities which may occur in equations (61, 62). This would correspond to amplitude phase monopulse, and some classification schemes[3] would fail in this case.

Equation (63) was discussed in a proceeding section, section III.

From equation (52),

$$\text{Ln}\left[\frac{((A-B) \cdot (A-B))^{1/2}}{((A+B) \cdot (A+B))^{1/2}}\right] = \text{Ln}\left[\left(\frac{\cosh(2\psi_o(u)) - \cos(2\phi_o(u))}{\cosh(2\psi_o(u)) + \cos(2\phi_o(u))}\right)^{1/2}\right] \tag{64}$$

Note that the greater $\psi_o(u)$, the less the amplitude. The signal would vary between the lower boundary

$$\text{Ln}\left[/\tanh\psi_o(u)/\right] \quad , \tag{65}$$

and the upper boundary

$$\text{Ln}\left[/\text{ctnh}\psi_o(u)/\right] \quad , \tag{66}$$

Also note that zero crossings occur when

$$\cos(2\phi_o(u)) = 0 \quad . \tag{67}$$

Since

$$RB \cdot RB = B \cdot B \quad , \tag{68}$$

$$(\mathbf{A}+R\mathbf{B})\cdot(\mathbf{A}+R\mathbf{B}) = \mathbf{A}\cdot\mathbf{A} + 2\mathbf{A}\cdot R\mathbf{B} + \mathbf{B}\cdot\mathbf{B} \quad ; \tag{69}$$

and

$$(\mathbf{A}-R\mathbf{B})\cdot(\mathbf{A}-R\mathbf{B}) = \mathbf{A}\cdot\mathbf{A} - 2\mathbf{A}\cdot R\mathbf{B} + \mathbf{B}\cdot\mathbf{B} \quad . \tag{70}$$

From equations (20, 22, 38), one has

$$(\mathbf{A}+R\mathbf{B})\cdot(\mathbf{A}+R\mathbf{B}) = G^2 e^{2\psi_e(u)}\left[\cosh(2\psi_o(u)) + \sin(2\phi_o(u))\right] \quad , \tag{71}$$

and

$$(\mathbf{A}-R\mathbf{B})\cdot(\mathbf{A}-R\mathbf{B}) = G^2 e^{2\psi_e(u)}\left[\cosh(2\psi_o(u)) - \sin(2\phi_o(u))\right] \quad . \tag{72}$$

Taking the natural logrithm of the ratio of the magnitudes yields

$$\mathrm{Ln}\left[\frac{((\mathbf{A}+R\mathbf{B})\cdot(\mathbf{A}+R\mathbf{B}))^{1/2}}{((\mathbf{A}-R\mathbf{B})\cdot(\mathbf{A}-R\mathbf{B}))^{1/2}}\right] = \mathrm{Ln}\left[\left(\frac{\cosh(2\psi_o(u)) + \sin(2\phi_o(u))}{\cosh(2\psi_o(u)) - \sin(2\phi_o(u))}\right)^{1/2}\right] , \tag{73}$$

and in this case the zero crossings occur when

$$\sin(2\phi_o(u)) = 0 \tag{74}$$

Of course, the same boundaries, equations (65, 66), apply.

It is interesting to note that for

$$\psi_o(u) = 0 \quad , \tag{75}$$

$$\frac{((\mathbf{A}+R\mathbf{B})\cdot(\mathbf{A}+R\mathbf{B}))^{1/2}}{((\mathbf{A}-R\mathbf{B})\cdot(\mathbf{A}-R\mathbf{B}))^{1/2}} = \left(\frac{1 + \sin(2\phi_o(u))}{1 - \sin(2\phi_o(u))}\right)^{1/2} , \tag{76}$$

$$= \left(\frac{1 - \cos(2\phi_o(u) + \pi/2)}{1 + \cos(2\phi_o(u) + \pi/2)}\right)^{1/2} , \tag{77}$$

$$= \tan(\phi_o(u) + \pi/4) \quad , \tag{78}$$

a phase shift of 45 degrees when compared with equation (55).

VII. A VARIATION

What if Σ and Δ instead of $\mathbf{A}$ and $\mathbf{B}$ are available? Then equations (61, 62, 63) would be replaced by,

$$\mathrm{Ln}\left[(\Delta \cdot \Delta)^{1/2}\right] - \mathrm{Ln}\left[(\Sigma \cdot \Sigma)^{1/2}\right] \quad , \tag{79}$$

$$\mathrm{Ln}\left[((\Sigma - \mathbf{R}\Delta) \cdot (\Sigma - \mathbf{R}\Delta))^{1/2}\right] - \mathrm{Ln}\left[((\Sigma + \mathbf{R}\Delta) \cdot (\Sigma + \mathbf{R}\Delta))^{1/2}\right] \quad , \tag{80}$$

and

$$\mathrm{Ln}\left[((\Sigma + \Delta) \cdot (\Sigma + \Delta))^{1/2}\right] - \mathrm{Ln}\left[((\Sigma - \Delta) \cdot (\Sigma - \Delta))^{1/2}\right] , \tag{81}$$

respectively, sum difference amplitude phase monopulse.

Equation (79) is just the natural logarithm of equation (52), and equation (81) follows from equation (41, 42), that is,

$$\Sigma + \Delta = 2\mathbf{A} \quad , \tag{82}$$

and

$$\Sigma - \Delta = 2\mathbf{B} \quad . \tag{83}$$

This leaves equation (80);

$$(\Sigma + \mathbf{R}\Delta) \cdot (\Sigma + \mathbf{R}\Delta) = \Sigma \cdot \Sigma + 2\Sigma \cdot \mathbf{R}\Delta + \Delta \cdot \Delta \quad , \tag{84}$$

and

$$\Sigma \cdot \mathbf{R}\Delta = (\mathbf{A} + \mathbf{B}) \cdot (\mathbf{R}\mathbf{A} - \mathbf{R}\mathbf{B}) \quad , \tag{85}$$

$$= -2\mathbf{A} \cdot \mathbf{R}\mathbf{B} \quad , \tag{86}$$

and from equations (50, 51, 38)

$$(\Sigma + \mathbf{R}\Delta) \cdot (\Sigma + \mathbf{R}\Delta) = 2\, G^2\, e^{2\psi_e(u)} \left[\cosh(2\psi_o(u)) - \sin(2\phi_o(u))\right] \; . \tag{87}$$

Similarly

$$(\Sigma - \mathbf{R}\Delta) \cdot (\Sigma - \mathbf{R}\Delta) = \Sigma \cdot \Sigma - 2\Sigma \cdot \mathrm{R}\Delta + \Delta \cdot \Delta \quad , \tag{88}$$

$$= 2\, G^2\, e^{2\psi_e(u)} \left[\cosh(2\psi_o(u)) + \sin(2\phi_o(u))\right] \; . \tag{89}$$

From equations (87, 89) it is readily apparent that equation (80) is equivalent to equation (62), and the discussion in the previous section would equally apply to equations (79, 80, 81).

VIII. A B PHASE MONOPULSE - REVISITED

When $\psi_o(u)$ is large, the boundary equations (65, 66) for amplitude phase monopulse, equation (61, 62, 63, or 79, 80, 81), would present problems, while A B phase monopulse, equation (40), is independent of $\psi_o(u)$. Of course, A B amplitude monopulse, equation (26), could be used to remove cycle ambiguities in A B phase monopulse.

Taking the difference of equations (44, 45), one has

$$(\mathbf{A}+\mathbf{B})\cdot(\mathbf{A}+\mathbf{B}) - (\mathbf{A}-\mathbf{B})\cdot(\mathbf{A}-\mathbf{B}) = 4\mathbf{A}\cdot\mathbf{B} \qquad (90)$$

Similarly, the difference of equations (69, 70), is

$$(\mathbf{A}+\mathbf{RB})\cdot(\mathbf{A}+\mathbf{RB}) - (\mathbf{A}-\mathbf{RB})\cdot(\mathbf{A}-\mathbf{RB}) = 4\mathbf{A}\cdot\mathbf{RB} \qquad (91)$$

the ratio of equations (90, 91) is

$$\frac{(\mathbf{A}+\mathbf{RB})\cdot(\mathbf{A}+\mathbf{RB}) - (\mathbf{A}-\mathbf{RB})\cdot(\mathbf{A}-\mathbf{RB})}{(\mathbf{A}+\mathbf{B})\cdot(\mathbf{A}+\mathbf{B}) - (\mathbf{A}-\mathbf{B})\cdot(\mathbf{A}-\mathbf{B})} = \tan\phi_o(u) \quad , \qquad (92)$$

and solving

$$\phi_o(u) = \arctan\left[\frac{(\mathbf{A}+\mathbf{RB})\cdot(\mathbf{A}+\mathbf{RB}) - (\mathbf{A}-\mathbf{RB})\cdot(\mathbf{A}-\mathbf{RB})}{(\mathbf{A}+\mathbf{B})\cdot(\mathbf{A}+\mathbf{B}) - (\mathbf{A}-\mathbf{B})\cdot(\mathbf{A}-\mathbf{B})}\right] \qquad (93)$$

The advantage of equation (92) over amplitude phase monopulse is that it is possible to solve for $\phi_o(u)$, and given $\phi_o(u)$, for u.

The disadvantage is that equations (90, 50, 51) yield

$$\begin{aligned} 4\,\mathbf{A}\cdot\mathbf{B} = {} & 2\,G^2\,e^{\psi_e(u)}\left[\cosh(2\psi_o(u)) + \cos(2\phi_o(u))\right] \\ & - 2\,G^2\,e^{\psi_e(u)}\left[\cosh(2\psi_o(u)) - \cos(2\phi_o(u))\right] \quad . \end{aligned} \qquad (94)$$

For $\psi_o(u)$ large, one would have the difference of two nearly equal quantities and this would be sensitive to noise.

Similarly, equations (91, 71, 72),

$$\begin{aligned} 4\,\mathbf{A}\cdot\mathbf{RB} = {} & 2\,G^2\,e^{2\psi_e(u)}\left[\cosh(2\psi_o(u)) + \sin(2\phi_o(u))\right] \\ & - 2\,G^2\,e^{2\psi_e(u)}\left[\cosh(2\psi_o(u)) - \sin(2\phi_o(u))\right] \end{aligned} \qquad (95)$$

which has the same noise problem.

To use A B amplitude monopulse, equation (26), to remove cycle ambiguity, $\psi_o(u)$ need not be large, but there would be a design tradeoff between ambiguity resolution and noise.

Instead of envelope detection to form these dot products, another possibility is coherent detection with digital computation. The desired dot products would be computed from the components of the vectors.

Of course, G and $\phi_e(u)$ must be eliminated since they are dependent upon the range to the target. From equations (20, 22),

$$(A \cdot A)^{1/2} \, (B \cdot B)^{1/2} = G \, e^{\psi(u)} \quad G \, e^{\psi(-u)} \quad , \tag{96}$$

$$= G^2 \, e^{\psi(u) \; + \; \psi(-u)} \quad , \tag{97}$$

$$= G^2 \, e^{2\psi_e(u)} \quad . \tag{98}$$

With equations (34, 38),

$$\frac{A \cdot B}{(A \cdot A)^{1/2} \, (B \cdot B)^{1/2}} = \cos(2\phi_o(u)) \quad , \tag{99}$$

and

$$\frac{A \cdot RB}{(A \cdot A)^{1/2} \, (B \cdot B)^{1/2}} = \sin(2\phi_o(u)) \quad . \tag{100}$$

Equations (99, 100) would then be used in a routine like FORTRAN's ATAN2 which yield the angle's quadrant.

IX. CONCLUSIONS

"There is no one best way."

Rhodes mentioned in the preface to Introduction to Monopulse[1] that he adopted the philosophy of Sir Robert Watson-Watts, "Cult of the Third Best: The Best Never Comes and the Second Best Comes Too Late."

There seems to have been a tendency in some of the monopulse literature to force all monopulse to fit Rhodes' "third best" theory monopulse. The fault lies not with Rhodes but with those who just accepted his initial treatment as final. Instead of dealing with unquestioned answers, attention should be given to unanswered questions.

A natural extention to this modification would be to incorporate polarization via Jones vectors, two diamensional vectors whose components are complex.

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2. McLendon, Robert, and Turner, Charles, Broadband Sensors for Lethal Defense Suppression, Microwave J., Sep 1983, pp. 85-102.

3. Sherman, Samuel M., Monopulse Principles and Techniques, Dedham: Artech House, 1984.

APPENDIX

P(u) Versus $G\,e^{\psi(u)}$

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P(u) Versus G $e^{\psi(u)}$

From equations (9, 10)

$$P(u)\,P(-u) = G^2\,e^{2\psi_e(u)} \tag{A-1}$$

and

$$\frac{P(u)}{P(-u)} = e^{2\psi_o(u)} \tag{A-2}$$

From an identity for the hyperbolic tangent,

$$\tanh\psi_o(u) = \frac{e^{2\psi_o(u)} - 1}{e^{2\psi_o(u)} + 1} \quad , \tag{A-3}$$

and equation (A-2), one has,

$$\tanh\psi_o(u) = \frac{\dfrac{P(u)}{P(-u)} - 1}{\dfrac{P(u)}{P(-u)} + 1} \tag{A-4}$$

or

$$\tanh\psi_o(u) = \frac{P_o(u)}{P_e(u)} \quad . \tag{A-5}$$

The right had sides of equations (A-4, A-5) will be familiar to those acquainted with Rhodes' Theory of Monopulse[1], and the left hand side maybe substituted where appropriate. That $\tanh\psi_o(u)$ and $\tan\phi_o(u)$ are opposite sides of the same coin is pleasing, if not revealing.

The hyperbolic cosine occurs extensively in the main body of this report. From its definition,

$$\cosh(2\psi_o(u)) = \frac{e^{2\psi_o(u)} + e^{-2\psi_o(u)}}{2} \quad , \tag{A-6}$$

and equation (A-2),

$$\cosh(2\psi_0(u)) = 1/2 \left(\frac{P(u)}{P(-u)} + \frac{P(-u)}{P(u)} \right) , \qquad \text{(A-7)}$$

or

$$\cosh(2\psi_0(u)) = \frac{P_e^2(u) + P_o^2(u)}{P_e^2(u) - P_o^2(u)} . \qquad \text{(A-8)}$$

Those who favor Rhodes' notation, P(u), could substitute the right hand sides of equations (A-7, A-8) where appropriate throughout this report.

Though the hyperbolic sine occurred only once in this report, equation (15), for completeness,

$$\sinh(2\psi_0(u)) = 1/2 \left(\frac{P(u)}{P(-u)} - \frac{P(-u)}{P(u)} \right) , \qquad \text{(A-9)}$$

or

$$\sinh(2\psi_0(u)) = \frac{2\ P_e(u)\ \ P_o(u)}{P_e^2(u) - P_o^2(u)} . \qquad \text{(A-10)}$$

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